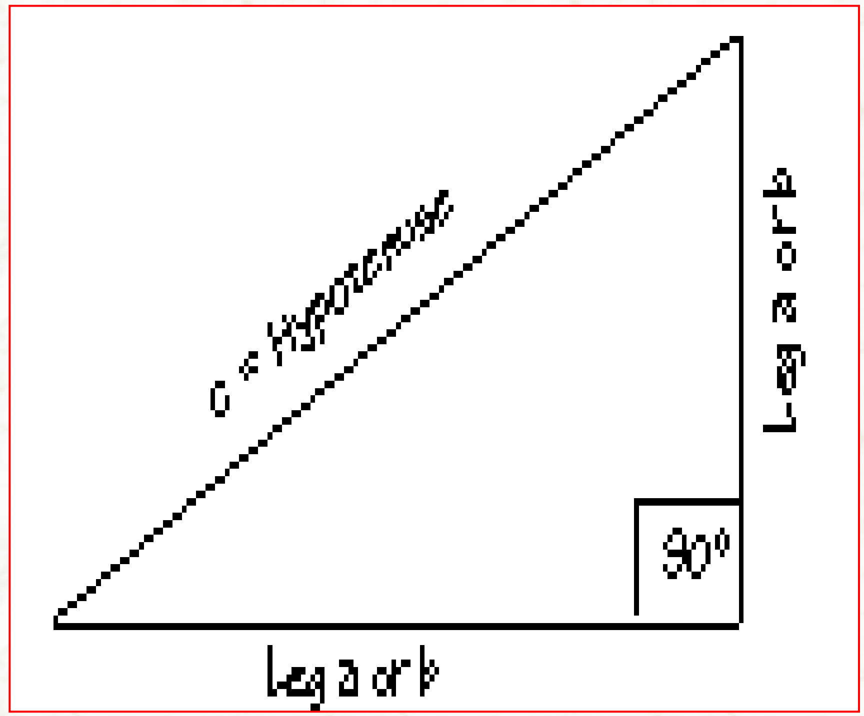
The Pythagorean Theorem

Pythagoras

- Lived in southern Italy during the sixth century B.C.
- Considered the first true mathematician
- Used mathematics as a means to understand the natural world
- First to teach that the earth was a sphere that revolves around the sun

Right Triangles

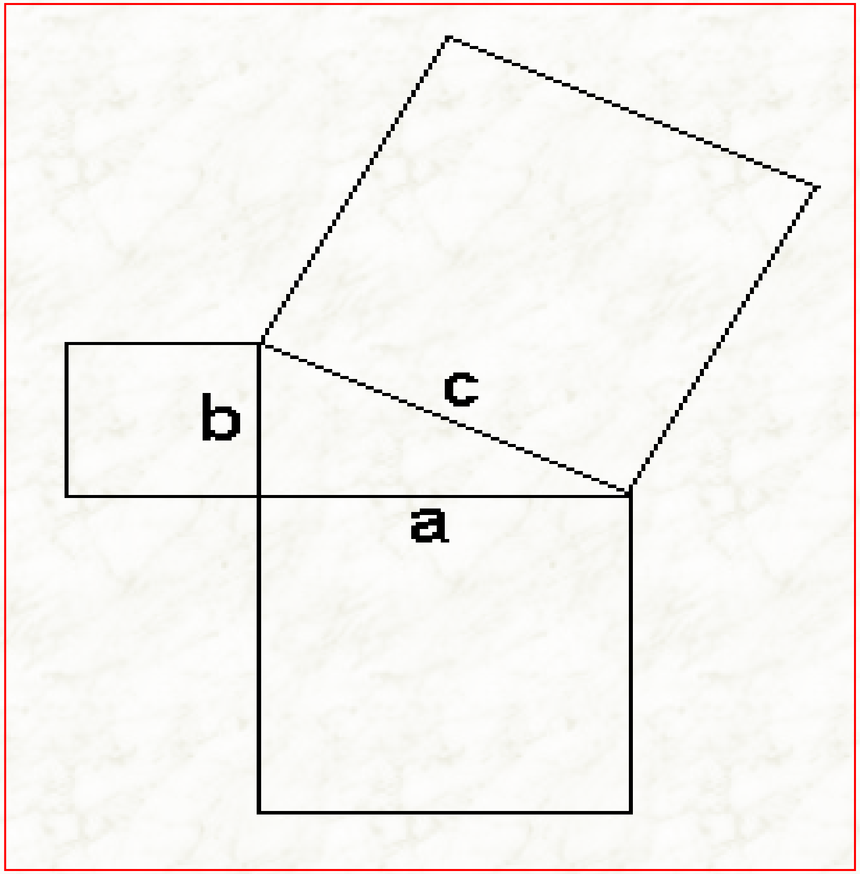
- Longest side is the hypotenuse, side c (opposite the 90º angle)
- The other two sides are the legs, sides a and b
- Pythagoras developed a formula for finding the length of the sides of any *right* triangle



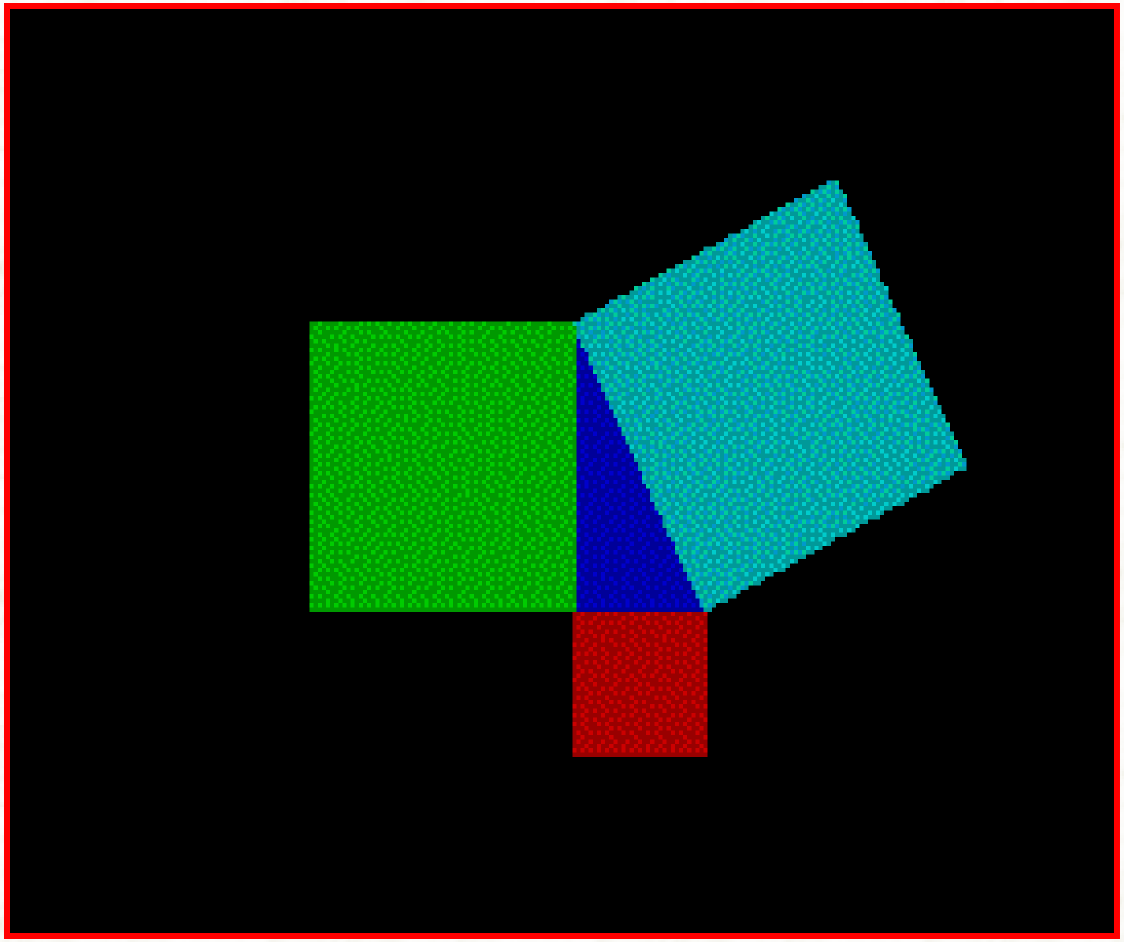
The Pythagorean Theorem

"For any right triangle, the sum of the areas of the two small squares is equal to the area of the larger."

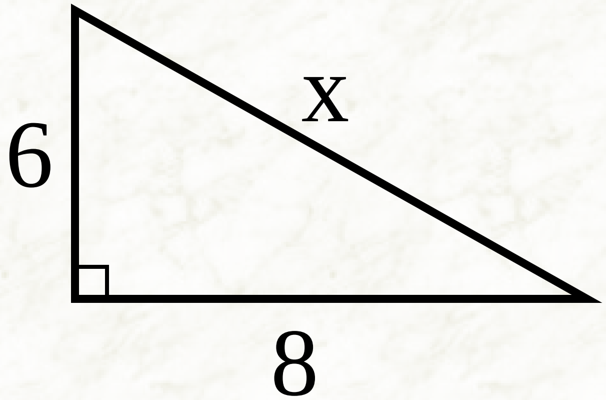
$a^2 + b^2 = c^2$



Proof



Solve for x.



$$a^2 + b^2 = c^2$$

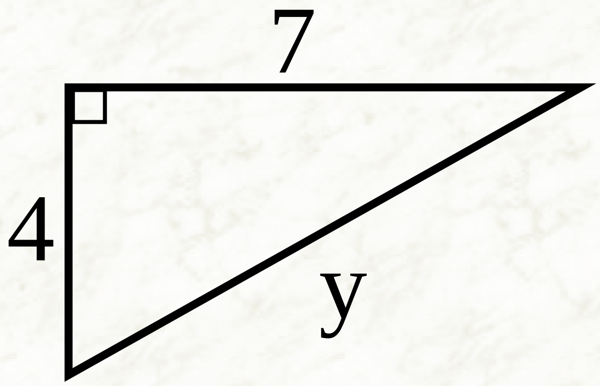
$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$100 = x^2$$

$$\sqrt{100} = \sqrt{x^2}$$

Solve for y.



$$a^2 + b^2 = c^2$$

$$7^2 + 4^2 = y^2$$

$$49 + 16 = y^2$$

$$65 = y^2$$

$$\sqrt{65} = \sqrt{y^2}$$

$$y \approx 8.1$$

Solve for t.

$$a^2 + b^2 = c^2$$

$$t^2 + 6^2 = 15^2$$

$$t^2 + 36 = 225$$

$$-36 \quad -36$$

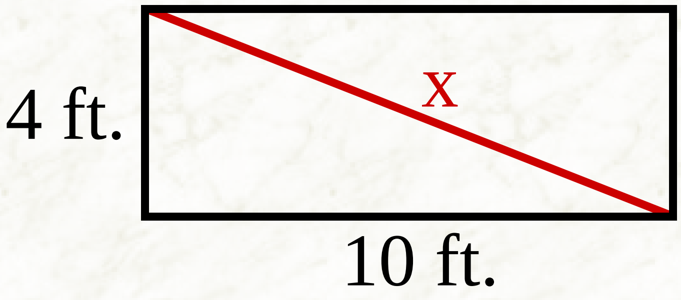
$$t^2 = 189$$

$$\sqrt{t^2} = \sqrt{189}$$

$$t = \sqrt{189}$$

$$t \approx 13.7$$

To the nearest tenth of a foot, find the length of the diagonal of a rectangle with a width of 4 feet and a length of 10 feet.



$$a^2 + b^2 = c^2$$

$$4^2 + 10^2 = x^2$$

$$16 + 100 = x^2$$

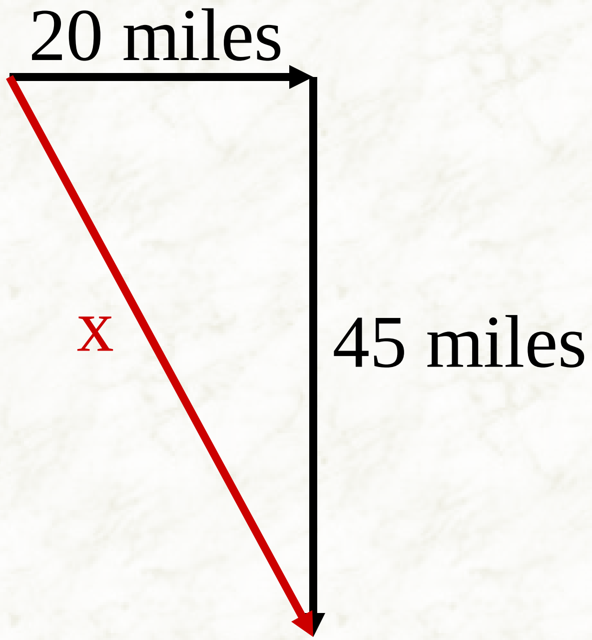
$$116 = x^2$$

$$\sqrt{116} = \sqrt{x^2}$$

$$x = \sqrt{116}$$

$$x \approx 10.8$$

A car drives 20 miles due east and then 45 miles due south. To the nearest hundredth of a mile, how far is the car from its starting point?



$$a^2 + b^2 = c^2$$

$$20^2 + 45^2 = x^2$$

$$400 + 2025 = x^2$$

$$2425 = x^2$$

$$\sqrt{2425} = \sqrt{x^2}$$

$$x = \sqrt{2425}$$

$$x \approx 49.24$$

Applications

- **The Pythagorean theorem has far-reaching ramifications in other fields (such as the arts), as well as practical applications.**
- **The theorem is invaluable when computing distances between two points, such as in navigation and land surveying.**
- **Another important application is in the design of ramps. Ramp designs for handicap-accessible sites and for skateboard parks are very much in demand.**

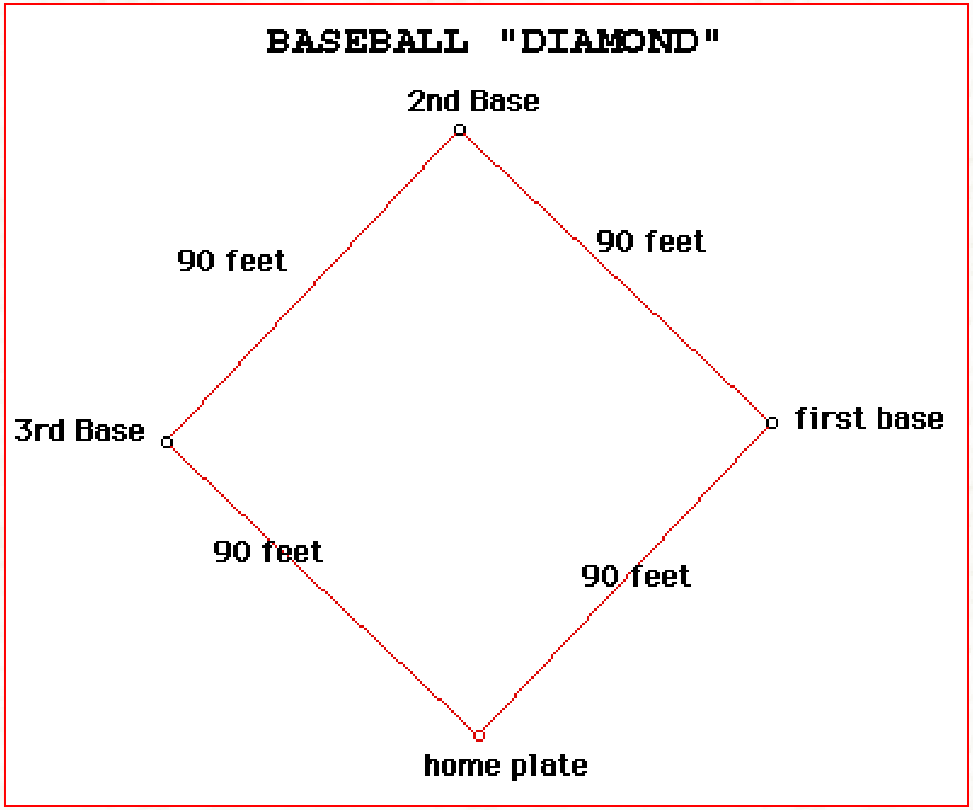
Baseball Problem

A baseball "diamond" is really a square.

You can use the Pythagorean theorem to find distances around a baseball diamond.

Baseball Problem

The distance between consecutive bases is 90 feet. How far does a catcher have to throw the ball from home plate to second base?



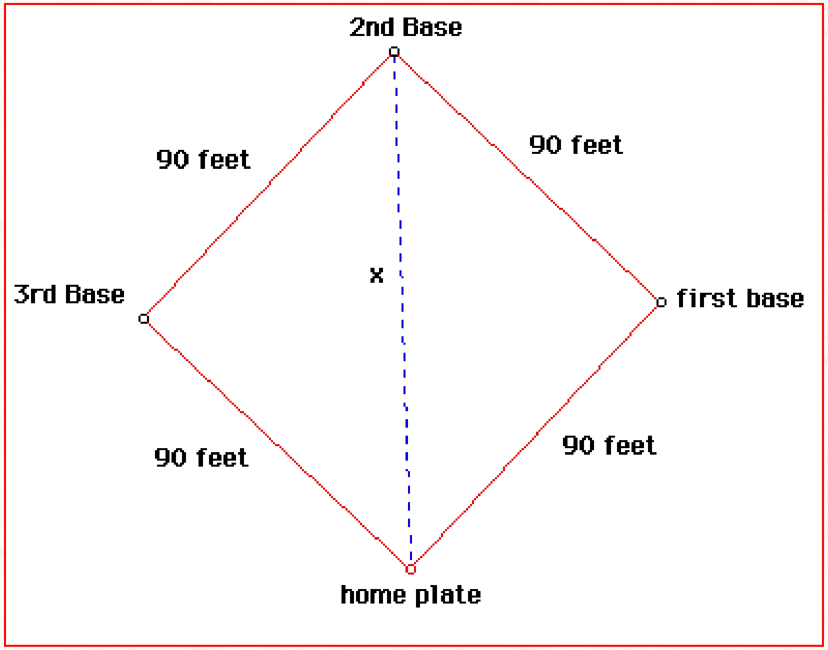
Baseball Problem

To use the Pythagorean theorem to solve for x, find the right angle.

Which side is the hypotenuse?

Which sides are the legs?

Now use: $a^2 + b^2 = c^2$

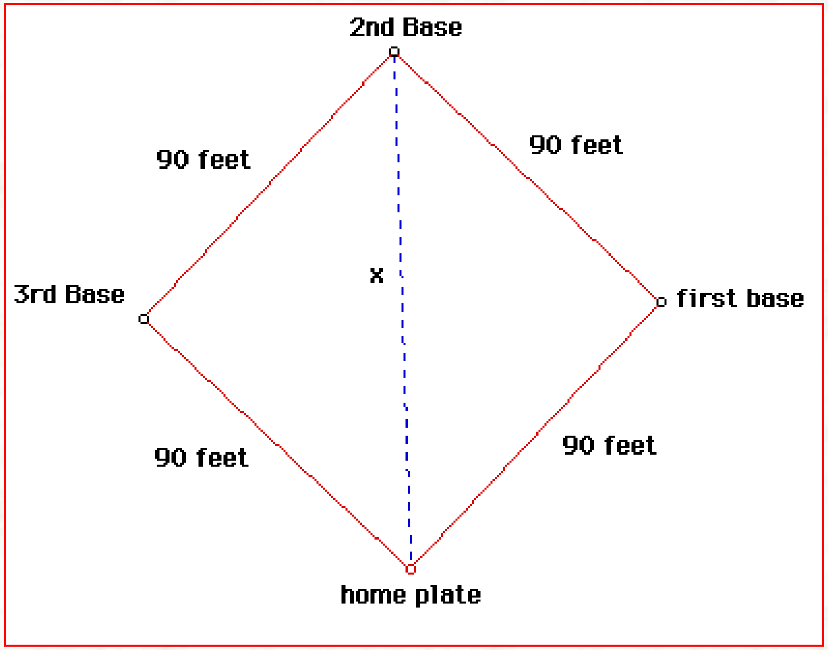


Baseball Problem
Solution

- The hypotenuse is the distance from home to second, or side x in the picture.
- The legs are from home to first and from first to second.
- Solution:

  $x^2 = 90^2 + 90^2 = 16,200$

  $x = 127.28$ ft



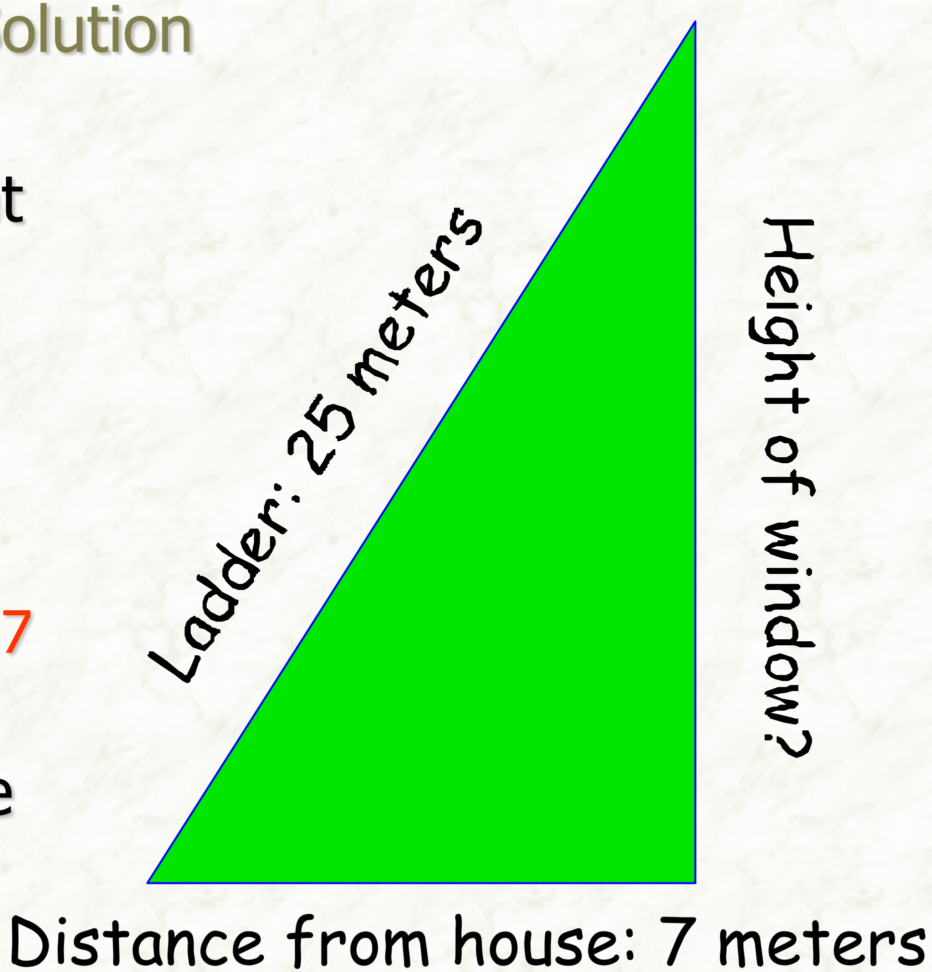
Ladder Problem

A ladder leans against a second-story window of a house.
If the ladder is 25 meters long,
and the base of the ladder is 7 meters from the house,
how high is the window?

Ladder Problem

Solution

- First draw a diagram that shows the sides of the right triangle.
- Label the sides:
  - Ladder is 25 m
  - Distance from house is 7 m
- Use $a^2 + b^2 = c^2$ to solve for the missing side.



Ladder Problem

Solution

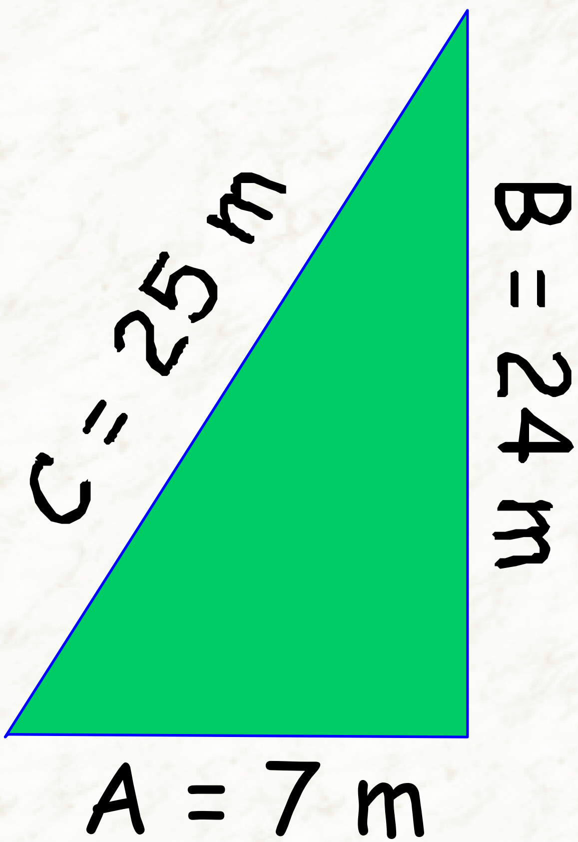
$7^2 + b^2 = 25^2$

$49 + b^2 = 625$

$b^2 = 576$

$b = 24$ m

How did you do?



Sources

Great info on the Pythagorean theorem, Pythagoras, and other math-related topics:

- The Baseball Problem
- Pythagoras of Samos
- Pythagoras Playground
- Microsoft Encarta 2000